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Specification and Drawings, as originally filed, with Application for Patent Serial No: **2,263,056,** on February 26, 1999, by **CERTICOM CORP.**, assignee of Robert Lambert, Robert Gallant, Ronald Mullin and Scott Vanstone, for "Method and Apparatus for Finite Field Basis Conversion".

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ABSTRACT

A method and systems provided for basis conversion in a cryptographic system. The method comprises the steps of a first correspondent transmitting an element represented in the first basis to an intermediate processor, the intermediate processor converting the element into a second basis representation and forwarding the converted element to the first correspondent who then uses the converted element in a cryptographic operation. A further embodiment of the invention provides for the intermediate processor to perform the basis conversion on a field element and then forward the converted element to a second correspondent. A still further embodiment of the invention provides for the correspondents in a cryptographic scheme making use of a bit string as a function of a sequence of traces of a field element, wherein the bit string is a shared secret for performing certain cryptographic operations.

METHOD AND APPARATUS FOR FINITE FIELD BASIS CONVERSION

The present invention relates to cryptographic systems and more particularly, to the conversion of elements in a finite field having one basis to elements of a finite field having another basis and wherein the elements are used in a cryptographic operation.

BACKGROUND OF THE INVENTION

Cryptographic operations are generally implemented on elements in a finite field. Various finite fields are of interest to cryptographers for example, the multiplicative groups of prime fields $F(p)$, the multiplicative group of finite fields of characteristic two, $F(2^n)$ and elliptic curve groups over finite fields, $E(F_p)$ or $E(F_{2^n})$. The elements in a given finite field are represented in terms of a basis for the finite field. The bases are also elements of the finite field.

Certain efficiencies may be realized in cryptographic operations by choosing a particular set of bases for that finite field. For example, in the finite field $F(2^n)$, two common choices of bases of the polynomial basis and a normal basis. A problem arises though in the choice of basis since communication between the two parties, although using the same cryptographic scheme but having different bases elements, requires the parties to perform a basis conversion operation on the field elements in order to obtain the same cryptographic result.

In general, if we let $F(q^n)$ be a finite field, where q is a prime or a prime power, the degree of the field is n and its order is q^n. A basis for the finite field is a set of n elements b_0, $b_1,\ldots b_{m-1} \in F(q^n)$ such that every element A of the finite field can be represented uniquely as a linear combination of basis elements:

$$A = \sum_{i=0}^{n-1} a_i b_i$$

where the $a_i \in F(q)$ are the coefficients. Arithmetic operations are then performed on this ordered set of coefficients.

It may be seen then generally that by using a different basis, a different ordered set of coefficients is used.

Various techniques have been implemented to convert between two choices of basis for a finite field. A conventional approach involves using a matrix multiplication, wherein basis conversion is performed using a change of basis matrix m, resulting in a matrix of size m^2. If m is typically 160 bits, then this occupies significant storage in devices such as a smart card. General finite field techniques are described in the "Handbook of Applied Cryptography", CRC Press, 1996 by S.A. Vanstone et al and incorporated herein by reference. Other techniques for basis conversion are described in United States Patent No. 5,854,759 to Kaliski et al, also incorporated herein by reference.

SUMMARY OF THE INVENTION

The present invention seeks to provide a method and apparatus for basis conversion, that is generally efficient in terms of memory and computation time and is particularly adapted for use with smart cards and other low power cryptographic tokens.

In accordance with this invention, there is provided a method for basis conversion, the method comprising the steps of a first correspondent transmitting an element represented in a first basis to an intermediate processor; the intermediate processor converting the element into a second basis representation; forwarding said converted element to the first correspondent; and the first correspondent operating on the converted element in a cryptographic operation.

BRIEF DESCRIPTION OF THE DRAWINGS

These and other features of the preferred embodiments of the invention will become more apparent in the following detailed description in which reference is made to the appended drawings wherein:

Figure 1 is a schematic diagram of an embodiment of a basis conversion system in accordance with the present invention;

Figure 2 is a schematic diagram of a further embodiment of a basis conversion system in accordance with the present invention; and

Figure 3 is a flow diagram illustrating a key exchange scheme in accordance with an embodiment of the invention.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

Referring to figure 1, a method according to a first embodiment of the invention is shown generally by numeral 10. In this embodiment, a pair of correspondents are represented by *A* and *B* and an intermediate processor, such as a server, certifying authority or other helper processor, is represented by *H*. It is assumed the correspondents *A* and *B* include processors for performing cryptographic operations and the like. Specifically, *A* and *B* perform cryptographic operations n a basis β_1 and β_2, respectively. It is further assumed that the respective cryptographic parameters are contained within the entities *A* and *B*. For example in an elliptic curve scheme the system parameters include at least a point *P* on the elliptic curve, the order of the curve and the parameters of the elliptic curve equation E.

In this embodiment, the entities *A* and *B* generates a respective random value k_i, generally the private session key and each compute a public value *kP*, represented in terms of their respective bases β_1 and β_2. One of the entities, A for example, transmits its public key $kP_{\beta 1}$ to the server *H*. The server *H* performs a basis conversion utilizing one of many basis conversion algorithms to convert the public key $kP_{\beta 1}$ represented in basis β_1 to a public key $kP_{\beta 2}$ represented in terms of the basis β_2. The converted key is transmitted back to the correspondent A. The correspondent A then computes signature $s = k^{-1}(h(m) + dr)$, where $r = kP_{\beta 2}$. The signature s and r are then transmitted to the other correspondent B, which is then processed by B in the basis β_2. Similarly if correspondent B wishes to communicate with A it also transmits its public key $kP_{\beta 2}$ to the server, which performs the conversion on the key and sends it back to the correspondent B. The correspondent B also computes a signature using $r = kP_{\beta 1}$.

In this embodiment, a helper or an intermediate processor is utilized to perform the basis conversion. Furthermore the cryptographic scheme is not compromised since the public key may be transmitted in the clear, without requiring a secure communication path between the correspondent and the server.

Referring to figure 2, a second embodiment according to the invention, is shown generally by numeral 20. In this embodiment, each of the correspondents A and B have a respective public key aP represented in terms of basis β_1 and bP represented in terms of basis β_2. The first correspondent A transmits its public key aP to the server H which performs the basis conversion on the element to a representation basis β_2 and transmits this key aP_{b2} to the second

correspondent B. The second correspondent B also transmits its public key bP_{b2} to the server where a basis conversion is performed on the key to the basis β_1 of the first correspondent. The key $bP_{\beta 1}$ is forwarded to the first correspondent A. Each of the correspondents then compute a common key by combining its private key with the other correspondents received public key. Thus, A computes $abP_{\beta 1}$ and B computes $baP_{\beta 2}$.

The correspondents have now performed a key exchange, each having a shared key, and only one of the correspondents need perform a basis conversion. The keys may then be used in subsequent steps of the encryption scheme.

In a third embodiment, again it is assumed that the correspondents A and B operate in bases β1 and β2 respectively. The bases β1 and β2 may represent any basis. Furthermore, we define a field element α such that correspondent A represents the element α in terms of the basis β1 and correspondent **B** represents the field element in terms of basis β2. The correspondents make use of a bit string that is a function of a sequence of traces of the field element as a shared secret to perform the certain cryptographic operations.

In this embodiment if we let p be a prime and let $q = p^m$, where $m \geq 1$. Let F_q be the finite field having q elements and Fq^n, the n-dimensional extension. The cyclic group **G** of Fq^n over Fq is generated by the mapping $\sigma(\alpha) = \alpha^q$, $\alpha \in Fq^n$, and is of order n. We may then define the trace function of Fq^n over F_q as

$$Tr_{F_{q^n}|F_q}(\alpha) = \sum_{\eta \in G} \eta(\alpha) = \sum_{i=0}^{n-1} \alpha^{q^i} .$$

For brevity, the trace function is simply represented as Tr. In the method of the present invention we make use of the property that the traces $Tr(\alpha_{\beta 1}) = Tr(\alpha_{\beta 2})$, that is the traces of an element α represented in terms of a basis β1 is the same as the trace of the element represented in terms of basis β2.

If a key of length n = 128 bits is to be constructed, then the traces of odd powers of α are taken. The traces, namely $Tr(\alpha)$, $Tr(\alpha^3)$, ...$Tr(\alpha^{257})$, are either 0 or 1. Since the trace is independent of the representation and it does not matter, which one of the entities performs the trace. As an aside it may be noted that we could also use the trace $Tr(f_1(\alpha))...Tr(f_k(\alpha))$ that is the

trace of $F(2^n)$ maps to the elements [0,1] or F(2). Therefore, f_i maps $F(2^n)$ to F (2). In general, any invariant function may be utilized for the trace.

In general if $F(q^n)$ is the finite field and F(q) is the ground field over which it is defined, the elements of the finite field can be represented in a number of ways depending on the choice of basis. Two common types of basis are polynomial basis and normal basis. If $\beta 1$ is a polynomial basis, then the basis elements may be represented as $1, \beta, \beta^2, \ldots \beta^{n-1}$, where β is a root or generator. Assuming the function $f(x) = 0$ and $f(x)$ is an irreducible of degree n i.e irreducible over the ground field . Then, if a field element is given by $\alpha = a_0 + a_1\beta^1 \ldots + a_{n-1}\beta^{n-1}$, the trace is given by

$$Tr(\alpha) = a_0 + a_1 Tr(\beta) + a_2 Tr(\beta^2) \ldots + a_{n-1}\, Tr(\beta^{n-1}).$$

It may be observed that the trace is linear and if the irreducible f(x) has the form $x^n + g(x)$ where the degree of g(x) is k, then

$Tr(\beta^j) = 0$ for $j = 1, 2 \ldots n\text{-}k\text{-}1$.

If the irreducible polynomial is given by

$$x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} \ldots + a_1$$

and if $a_{n-1} = 0$ then $Tr(\beta) = 0$, and $a_{n-1} = 0$ and $a_{n-2} = 0$ then $Tr(\beta^2) = 0$. The observation is that if consecutive coefficients of the field element α are zero then the trace of that number of terms is zero.

Thus, we may use the trace bit string as a shared secret to perform the remaining cryptographic operations. In deciding upon a key, the users (correspondents) normally select a bit string that is a function of a sequence of traces of a selected field element. For example if a bit string (key) of length 3 is desired, the trace of α, α^3, α^2 could be used. The order of the sequence of traces may on occasion be arbitrarily chosen but known to the correspondents. The following examples more clearly illustrate the derivation of a key.

Example1: In this example the trace of α and α^3 is used to create a binary key of length 2.

Basis 1: The irreducible chosen is $f(x) = x^3 + x + 1 = 0$; $x^3 = x + 1$

Element α in this basis is $\alpha = (1 + x^2)$ then the key = $(Tr(\alpha), Tr(\alpha^3))$

$Tr(1) = 1 + 1^2 + 1^4 = 1;$ $\quad (x^4 = x^2 + x)$

$Tr(x) = x + x^2 + x^4$

$= x + x^2 + x^2 + x = 0$

$Tr(x^2) = x^2 + x^4 + x^8$

$= x^2 + (x^2 + x) + (x^2 + x)^2$

$= x + (x^2 + x) + x^2 = 0$

$Tr(\alpha) = Tr(1+x^2) = Tr(1) + Tr(x^2) = 1 + 0 = 1$

$\alpha = \alpha.\alpha^2 = (1 + x^2)(1+x^2)^2 = (1 + x^2)(1 + x^4)$

$= (1 + x^2)(1 + x + x^2)$

$= 1 + x + x^2 + x^2 + x^3 + x^4$

$= 1 + x + x^3 + x^4$

$= 0 + x^2 + x$

$= x^2 + x$

$Tr(\alpha^3) = Tr(x^2) + Tr(x) = 0 + 0 = 0$

Thus the key = (1,0)

Example 2:In this example a different basis is used (basis 2) and α is converted to its representation in this basis by (1) finding a root r for the polynomial for basis 1 in the representation generated by basis 2, and (2) then evaluating the polynomial representing α in basis 1 at r. The traces of α and α^3 are calculated in basis 2 to generate the same binary key as was created in basis 1 above.

Basis 2: The irreducible chosen is $g(y) = y^3 + y^2 + 1$; $y^3 = y^2 + 1$

To find α in basis 2, find a root of $f(x) = x^3 + x + 1$ (the irreducible in basis 1) in basis 2.

Note: $(y+1)^3 + (y+1) + 1 = y^3 + y^2 + y + 1 + y + 1 + 1 = 0 + y + 1 + y + 1 = 0$

Let $r = y + 1$, then $\alpha = 1 + x^2 \rightarrow \alpha' = 1 + r^2 = 1 + (y+1)^2 = 1 + y^2 + 1 = y^2$

Key = $(Tr(\alpha'), Tr(\alpha')^3)$; $y^4 = y^3 + y = y^2 + y + 1$

$Tr(1) = 1 + 1 + 1$

$Tr(y) = y + y^2 + y^4 = y + y^2 + y^2 + y + 1 = 1$

$Tr(y2) = y^2 + y^4 + y^8 = y^2 + y^2 + y + 1 + (y^2 + y + 1)^2$

$= y + 1 + y^4 + y^2 + 1$

$= y^4 + y^2 + y$

$= y^2 + y + 1 + y^2 + y = 1$

$Tr(\alpha') = Tr(y^2) = 1$

$(\alpha')^3 = y6 = (y^3)^2 = (y^2 + 1)^2 = y^4 + 1 = y^2 + y + 1 + 1 = y^2 + y$

$Tr((\alpha')^3) = Tr(y^2 + y) = Tr(y^2) + Tr(y) = 1 + 1 = 0$

Thus the key = (1,0) as in basis 1.

Referring to figure 3, a key agreement scheme according to an embodiment of the invention is shown generally numeral 30. The correspondents A and B operate in bases β1 and β2 respectively. The bases β1 and β2 may represent any basis. Furthermore A and B each have the following system parameters, a long term private key d and a long-term public key $Q_A = d_aP$ and $Q_B = d_bP$, where P is a point on an elliptic curve represented in terms of the respective bases. The correspondent **A** represents P in terms of the basis β1 and correspondent **B** represents P in terms of basis β2. In a typical Diffie-Hellman key agreement scheme, each of the correspondents A and B generate respective ephemeral private keys k_A and k_B and compute a corresponding short term (session) public keys $k_AP_{\beta 1}$ and $k_BP_{\beta 2}$. A and B exchange their respective public keys, and convert them to their own basis. If the correspondents are low power devices, such as smart cards or the like, then basis conversion may be performed by an intermediate processor such as described with reference to figures 1 and 2. Alternatively, if the correspondents have sufficient compiling power, then basis conversion may be performed by the correspondents themselves, according to one of many basis conversion methods. In any event, after the basis conversion, correspondent A has B's public key $(k_BP_{\beta 2})_{\beta 1}$ and B has A's public key $(k_AP_{\beta 1})_{\beta 2}$. A shared secret is computed in their respective basis by computing $k_A(k_BP_{\beta 2})_{\beta 1=\alpha\beta 1}$ and $k_B(k_AP_{\beta 1})_{\beta 2=\alpha\beta 2}$. Each of the correspondents takes a sequence of traces of their respective field element α to derive a common bit string.

Applying the method to a signature scheme, the correspondent A generates its ephemeral public session key $kP_{\beta 1}$. A trace sequence may be constructed, for example, of the x-coordinate of $kP_{\beta 1}$ producing a bit string T. The bit string is passed through a hash function g to derive a signature component r. A second signature component $s = k^{-1}(m + dr)$ is computed, where d is A's long term private key. The signature components are transmitted to B for verification. The verifier B computes $E'ms^{-1}\ P_{\beta 2}+rs\text{-}1Q_{A\ \beta 2},= kP_{\beta 2}$ where $Q_{A\ \beta 2}$ is the long term public key of A in basis 2. This basis conversion could be performed by A using an intermediate H as described earlier. B then generates a sequence on the computed value $kP_{\beta 2}$, and applies the hash function g to derive a value r'. If r'=r, then the signature is verified.

Although the invention has been described with reference to certain specific embodiments, various modifications thereof will be apparent to those skilled in the art without departing from the spirit and scope of the invention as outlined in the claims appended hereto.

THE EMBODIMENTS OF THE INVENTION IN WHICH AN EXCLUSIVE PROPERTY OR PRIVILEGE IS CLAIMED ARE DEFINED AS FOLLOWS:

1. A method for basis conversion between a pair of correspondents, said method comprising the steps of:

transmitting the element represented in a first basis from a first correspondent to an intermediate processor;

converting the received element into a second basis representation by said intermediate processor;

forwarding said converted element to the first correspondent; and

operating on said converted element by said first correspondent in a cryptographic operation.

2. In a cryptographic system, a method for generating a basis independent bit string, said method comprising the steps of:

representing a field element in terms of a first basis;

computing a function of a sequence of traces of said field element; and

using said sequence of traces as said bit string.

3. A method as defined in claim 2, including the step of using said bit string as a shared secret in said cryptographic scheme.

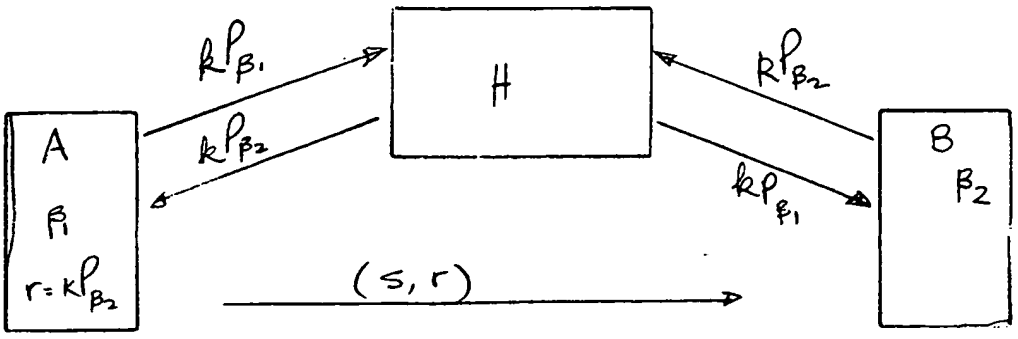


Figure 1

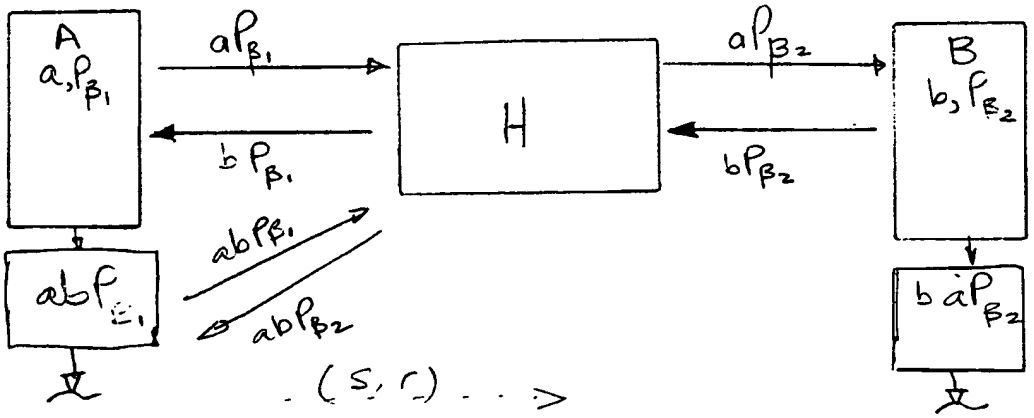
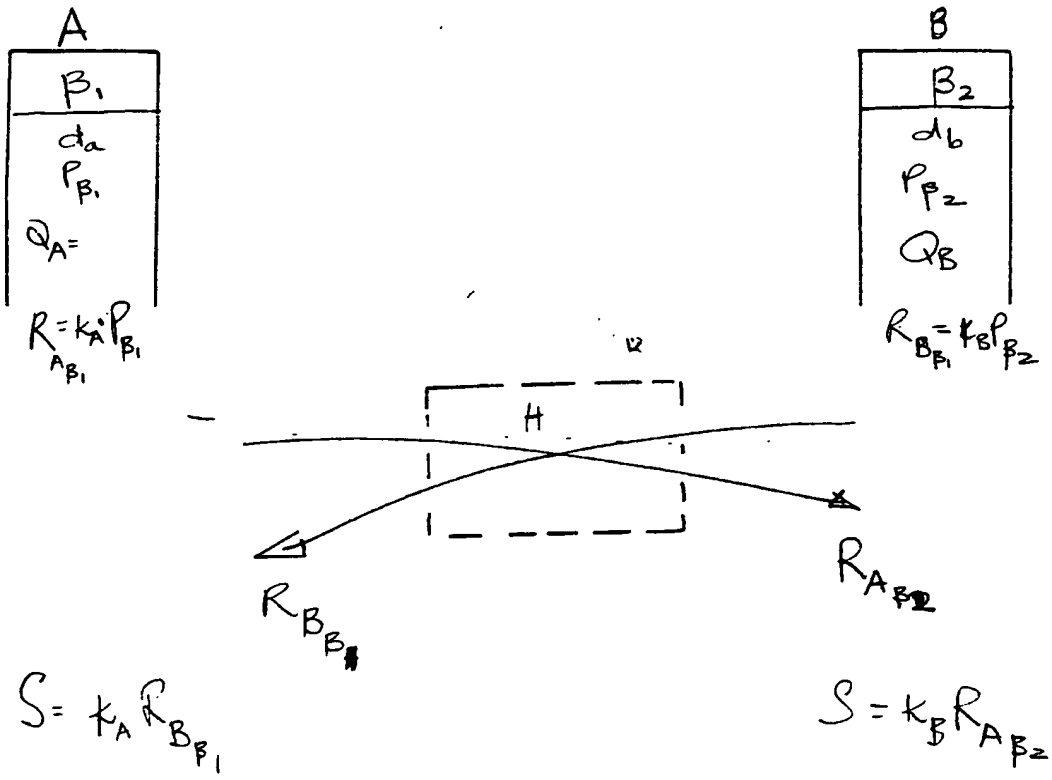


FIGURE 2



$T = [Tr(S), T(S^n) \ldots]$ ⟵ identical string ⟶ $T = [Tr(S), T(S^n) \ldots]$

Fig. 3